$$\square \quad = \quad \text{Constant x } \frac{\text{(W or P) x } L^3}{\text{E x } I}$$

Term	**Definition**	**Units**
□	Maximum **Deflection**	INCHES
Constant	Depends on **Loading** Configuration and **Support Condition**. Value of the constant is given in the pull-down reference tab on the ARE exam which comes from the AISC Manual. (get somewhat familiar with these. End of Section 2 in the AISC (Page 2-296… Green Manual) (also see cheat sheet I have posted on the ARE Forum)	None
W	This is the RESULTANT of a Uniform **Load** (Make sure you do NOT use w in #/FT in this equation) The AISC Manual lists the deflection in terms of wL^4 That version of the same equation is confusing as far as units ! It is easier to think of it as (w L) x L^3 or WL^3	(Pounds, Kips)
P	This is the magnitude of the Concentrated **Load**(s) applied on a beam	(Pounds, Kips)
L	The Span of the beam subjected to **loading**	(Feet to be converted to inches)
E	The Modulus of Elasticity of the **Material**	(PSI, KSI)
I	The Moment of Inertia of the Cross-Section **Geometry** $I = (bd^3)/12$ for a *rectangular* **shape** regardless of its material, It could be made of cardboard and it would still have the same properties of **Geometry** (b,d,A,S, I,...)	(in^4)

General Comments about the Deflection Equation:

It is very important to understand the way each term in this equation affects the resulting deflection. It is possible to get a question that requires you to calculate the deflection for a set of given conditions. In this case, the most important thing to get the correct answer is to keep your units straight. I cannot emphasize this enough.

Looking at this equation, the **Deflection, (□)** , MUST be expressed in INCHES and so we should NOT have FEET anywhere in this equation. This automatically implies that the **Span**, (**L**) , must be converted from FEET into INCHES BEFORE cubing the span… ie, $\cancel{(18')}^3$ instead you would use $(18\text{x}12=216")^3$. Likewise, use $(16\text{x}12)^3$ instead of $(\cancel{16}')^3$.

The second units-issue to be careful of in this equation is that (**E)** needs to have units similar to the Load **(W or P**) in the equation. If the load is in Pounds, then make sure that **E** is in

Pounds per Sq. In. (not Kips per Sq. In). E for steel (A-36 or A-50) is 29,000,000 PSI if the load (P or W) is in Pounds; otherwise, just express E = 29,000 KSI if (**W or P**) is in Kips. With lumber it is more typical to use Pounds for the loads and PSI for the Modulus of Elasticity, E . With Steel it is more typical to use Kips for (W or P) and KSI for E.

As far as the mechanics of using your calculator efficiently (and properly) to navigate the nasty math in this equation, I would suggest the following sequence of calculation,

- Start with the span **(L)**, convert it to inches by multiplying by 12, <u>THEN</u> cube the product, so 16' = $(192")^3$
- Multiply by **(W or P)** with the correct units depending on E
- Multiply by the Numerator of the **Constant** (for example the Constant for a uniformly loaded beam is 5 / 384, so multiply by 5; or if this is a concentrated load at midspan, then the constant is 1 / 48, so multiply by 1 (Please refer to AISC for these values)
- You have just finished multiplying every term in the Numerator by one another
- Next start dividing by every term in the Denominator one at a time
- So divide by the Denominator of the **Constant** next, ie, divide by 384 for a uniform load, 48 for a single concentrated load at midspan, etc….
- Divide by **E** (careful that the units of **E** match the units of **W or P**;; Pounds of Load matches **E** in PSI;; Kips of Load matches E in KSI!!). Otherwise convert Kips to Pounds by multiplying by 1000.
- Continue and divide by **I**
- Now you can finally press = and that should be your answer

So this sequence boils down to multiply everything in the Numerator, then divide by each term in the denominator, and finally presst =.

Ie… $(\text{Span in inches})^3$ **(x)** Load **(x)** Constant Numerator
(÷) Constant Denominator **(÷)** **E** **(÷)** **I** **= ANSWER**

- If your answer is too small (for example 0.00032 inches) chances are that you forgot to convert Kips to Pounds in the Load or **E** to match, or you did not convert the span into inches <u>BEFORE</u> cubing the span

More likely than not, though, for the ARE you will need to understand **qualitatively** the different factors and how they impact the deflection, rather than the tedious crunching of partially meaningless (to you) numbers. That being said, let's examine what each factor in that equation is and how it affects the deflection outcome.

- The **Constant** describes 2 issues:

    1) This term describes the support conditions of the beam (simply supported, continuous or cantilevered). Obviously cantilevered beams deflect significantly more than simply supported beams for the same span (**L**), loading configuration (**W or P**), the same material (**E**), and the same Geometry of section, ie, Moment of Inertia (**I**). For the same conditions, a continuous beam will deflect even less than a simply supported beam with the same conditions. These different support conditions are reflected in this **Constan**t.

    2) The type of load that the beam supports may typically be Uniformly applied over the span (**W**) or Concentrated at midspan (**P**) for example. For the same support condition, the same magnitude (if P=W in magnitude, theoretically), and the same span (**L**), material (**E**) and geometry (**I**); then the concentrated load (**P**) at midspan would create much more deflection as seen in the coefficient of (1/48), versus the deflection that results from the same load uniformly distributed (**W**) over the same span (5/384). Please do not freak out over these coefficients, they should be given to you. The reference pull-down tab on the exam has standard loading conditions that are reproduced from the AISC Manual and includes these constants. You need to remember that for the same total load P (assume P =24 K for the sake of illustration), concentrated in the middle creates more deflection than 2 concentrated loads of 12K each at 1/3 points along the span (still 24 K Total). Furthermore, 3 concentrated loads of 8K each applied at 1/4 points along the span (still 24 K Total) would create even less deflection than the two 12K loads, which in turn creates less deflection than the single 24K at midspan. Four loads of 6K each at fifth points along the span (still 24K) create a smaller deflection, and least of all a uniform load of 24K total would deflect the least.

- The **Load** is represented as a **Uniform** load (having a coefficient **w** in #/FT or K/FT or a Resultant **W** in # or Kips) or a **Concentrated** Load (**P**). Doubling the Load (either **W** or **P**) will cause twice the deflection if all other factors are held constant.

    <u>Example:</u> If you have a beam loaded with P= 24K concentrated load at midspan with a resulting deflection of 0.75”. How much will the same beam deflect if the load is reduced to 20K and the span remains the same?

    New Deflection (for 20K) = 0.75” x (20K/24K) = 0.625 “

    Ratio is simple to set up if no other variable is changing, Less load will result in “proportionally” less deflection

- If the **Span** (**L**) is doubled, then the resulting Deflection will be $2^3 = 8$ times as much as initially calculated for the shorter span. The slightest increase in the span increases the deflection **SIGNIFICANTLY.** Span is the single most important determinant of deflection (if all other factors are held constant)

  Example 1: If you have a beam loaded with a certain Load (Uniform or concentrated) on an 18' span that results in a 0.75" deflection. How much will the deflection become if the span is increased to 20' for the same load and configuration, the same beam geometry and material?

  New Deflection (for 20' span) = 0.75" x $(20 / 18)^3$ = 1.03"

  This is quite an increase in deflection for a mere 2' increase in length, and is resulting because the ratio of spans is being cubed!

  (Please note that the conversion from FEET to INCHES is not necessary in this case since both Numerator and Denominator will be changed by the same amount and will thus cancel out!)

  Example 2: If you have a beam loaded with a certain Load (Uniform or concentrated) on an 18' span that results in a 0.75" deflection. How much will the deflection become if the span is decreased to 14' for the same load and configuration, the same beam geometry and material?

  New Deflection (for 14' span) = 0.75" x $(14/18)^3$ = 0.35"
  (This is approx. half the initial deflection for a decrease of only 4' in span!

- Looking at variations in material stiffness or the Modulus of Elasticity(**E**) next, we find that Deflection is **inversely proportional** to the Modulus of Elasticity (**E**). Simply stated, the stronger the material, the higher its Modulus of Elasticity (**E**), and therefore, the smaller the resulting deflection, (If all other variables are held constant). Similarly, the weaker the material, the smaller its Modulus of Elasticity and the larger the resulting deflection for the same load (**W or P**), span (**L**) and geometry (**I**).

  Example 1: If you have a beam of a certain species of wood with E = 1.6×10^6 PSI, loaded with a certain Load (Uniform or concentrated) on a certain span (**L**) and results in a 0.75" deflection. How much will the deflection become if the material is replaced with another species of wood that has E = 1.4×10^6 PSI, for the same span, load and configuration, and the same beam geometry?

  New Deflection (for E = 1.4×10^6 PSI) = 0.75" x (1.6 / 1.4) = 0.86"

  The 10^6 term is omitted from the Numerator and Denominator since it cancels out

  I know that the above is hard to accept for many of you (I have been teaching this for long enough to know where the hiccups are!!!). You have to ignore the math for a bit and retain your logic! The stronger piece of lumber (E = 1.6) resulted in the 0.75" deflection. So when we replaced the first species with a "trashier" one (E=1.4), we should expect MORE deflection since the second species is, well ….. trashier!

So regardless of your “math” intuition, keep your wits about you, the answer should be more than 0.75”; so you cannot cross-multiply since these quantities are **inversely proportional!** The stronger the species of wood, the less the resulting deflection. The weaker the species of wood, the larger the resulting deflection.

Example 2: If you have a beam of a certain species of wood with E = 1.6×10^6 PSI, loaded with a certain Load (Uniform or concentrated) on a certain span (**L**) and results in a 0.75” deflection. How much will the deflection become if the material is replaced with another species of wood that has E = 1.8×10^6 PSI, for the same span, load and configuration, and the same beam geometry?

New Deflection (for E = 1.8×10^6 PSI) = 0.75” x (1.6 / 1.8) = 0.67”

Remembering your logic in setting up this proportion, the second species of lumber is stronger than the first (E = 1.8×10^6 vs. 1.6×10^6), and therefore, the second deflection should be smaller than 0.75”.

- Looking at changes in Moment of Inertia (**I**) next, we find that Deflection is also **inversely proportional** to Moment of Inertia. Simply stated, the deeper the member, the higher its Moment of Inertia (**I**), and therefore, the smaller the resulting deflection, (If all other variables are held constant). Similarly, the shallower the member, the smaller its Moment of Inertia (**I**), and the larger the resulting deflection for the same load (**W or P**), span (**L**) and material (**E**).

  Moment of Inertia (**I**) for a rectangular member is defined as $bx(d)^3 / 12$. An increase in width increases the Moment of Inertia, but an increase in depth will SIGNIFICANTLY increase the Moment of Inertia (thus markedly reducing the deflection).

Example 1: If you have a 4x10 beam, (assume actual dimensions, NOT Nominal, just for ease of calculation. I really don’t think you need to perform many calculations on the ARE if you can develop a good sense of what the numbers are telling you!) of a certain species of wood, loaded with a certain Load (Uniform or Concentrated) on a certain span (**L**) and results in a 0.75” deflection. How much will the deflection become if the 4x10 beam is replaced with a 4x12 beam of the same species of wood, for the same span, load and configuration?

We need to set up a proportion that relates the Moments of Inertia of each of the different sections.

For 4x10, **I** = $bx(d)^3 / 12 = 4 \times (10)^3 / 12 = 333.33$ in^4
For 4x12, **I** = $bx(d)^3 / 12 = 4 \times (12)^3 / 12 = 576$ in^4

So, for 4x10 with **I** = 333.33 in^4 and □ = 0.75”
For 4x12 with **I** = 576 in^4 how much is resulting deflection

New Deflection (for 4x12 with **I** = 576 in^4) =0.75” x (333.33 / 576) =0.43”

Example 2: If you have a 2x12 beam (assume actual dimensions, NOT Nominal) of a certain species of wood, loaded with a certain Load (Uniform or Concentrated) on a certain span (**L**) and results in a 0.75” deflection. How much will the deflection become if the 2x12 beam is replaced with a 2x10 beam of the same species of wood, for the same span, load and configuration?

We need to set up a proportion that relates the Moments of Inertia of each of the different sections.

For 2x12, $\mathbf{I} = bx(d)^3 / 12 = 2 \times (12)^3 / 12 = 288 \text{ in}^4$

For 2x10, $\mathbf{I} = bx(d)^3 / 12 = 2 \times (10)^3 / 12 = 166.67 \text{in}^4$

So, for 2x12 with $\mathbf{I} = 288 \text{ in}^4$ and □ = 0.75”

For 2x10 with $\mathbf{I} = 166.67 \text{ in}^4$ how much is resulting deflection?

New Deflection (for 2x10 with $\mathbf{I} = 166.67 \text{ in}^4$) =0.75”x(288 / 166.67) =1.3”